Study of Magnetoconvection Impact on a Solenoid Coil Cooling by Ferrofluid with a Spectral / Finite Element Method

Raphaël Zanella[1,3], Caroline Nore[1], Frédéric Bouillault[2], Loic Cappanera[3], Ignacio Tomas[3], Xavier Mininger[2], and Jean-Luc Guermond[3]

[1]LIMSI, CNRS, Univ. Paris-Sud, Université Paris-Saclay, F-91405 Orsay, France, zanella@limsi.fr, Caroline.Nore@limsi.fr
[2]GeePs, UMR 8507 CNRS / CentraleSupelec - Universités UPMC et UPSud, 91192 Gif sur Yvette cedex, France, Frederic.Bouillault@geeps.centralesupelec.fr, Xavier.Mininger@geeps.centralesupelec.fr
[3]Department of Mathematics, Texas A&M University, College Station, TX 77843 USA, cappanera@math.tamu.edu, itomas@tamu.edu, guermond@math.tamu.edu

The potential of magnetoconvection for transformer cooling by ferrofluid is numerically studied. A code combining spectral and finite element method is applied on a solenoid coil system. Magnetostatic, Navier-Stokes and energy equations are solved simultaneously. A vegetable oil seeded with magnetite nanoparticles at a volume fraction of 7 % is considered. The magnetization of the ferrofluid is function of temperature through an approximation of classical Langevin's law. Magnetic and temperature fields are used to update the magnetic action, modeled by the Kelvin force, on the ferrofluid momentum at each time step. With the chosen parameters, a steady regime is reached and the solution is axisymmetric, for regular oil and ferrofluid cooling. The comparison shows that the magnetoconvection modifies the flow convection pattern and speed. The temperature increase in the coil is consequently reduced by about 11 % with ferrofluid cooling.

***Index Terms*—coupled problem, finite element, magnetoconvection, spectral method, transformer cooling.**

I. Introduction

A FERROFLUID is a stable suspension of magnetic nanoparticles in a non magnetic liquid carrier. Typically, the nanoparticles are made of magnetite and the carrier is water or oil. A magnetic field transmits momentum to the ferrofluid through the nanoparticles. The literature often mentions the Kelvin force model (in N/m^3) [1],

$$\mathbf{F} = \mu_0(\mathbf{M}\cdot\nabla)\mathbf{H}, \tag{1}$$

where μ_0 is the vacuum magnetic permeability, $\mathbf{M}$ the magnetization and $\mathbf{H}$ the magnetic field.

The dependence of the magnetization with respect to temperature can lead to magnetoconvection when magnetic field and temperature gradients are combined. It is natural to think about immersed transformer cooling as a possible application. If the heat transfer rate is increased because of the nanoparticles, the volume of cooling fluid could be reduced, or mechanical cooling systems avoided. Moreover, instead of conventional mineral oil, the liquid carrier could be substituted by vegetable oil, with higher viscosity but biodegradable and non toxic.

In [2-3], the authors obtained encouraging results on transformer cooling by ferrofluid. In this work, numerical developments are integrated to the SFEMaNS code [4], based on spectral and finite element method, to solve magnetostatic, momentum and energy equations for ferrofluids. The ferrofluid magnetization follows an approximation of Langevin's law, generally considered to include the effects of temperature dependence [5]. Being a function of temperature and magnetic field, the magnetic force is updated at each time step. Computing the temperature in the solid regions allows to measure the impact of magnetoconvection on the coil heating.

II. Modeling

An electromagnetic system constituted of a double copper coil crossed by a DC current and immersed in a vegetable oil based ferrofluid is considered. The PVC tank's walls are modeled as well, as presented in Fig. 1.

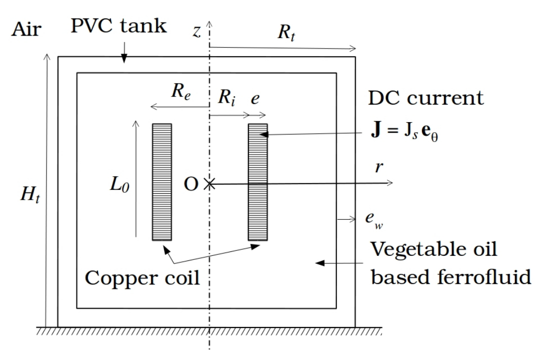

Fig. 1. Problem setup.

Quasi-steady regime approximation for electromagnetism is used. The ferrofluid is a continuum medium with Newtonian fluid behavior. Boussinesq approximation is used and viscous dissipation is neglected. The ferrofluid magnetization is assumed to be instantaneously aligned with the magnetic field. Finally, the transient term containing the pyromagnetic coefficient in the energy equation of [1] is also neglected.

The magnetostatic equations are

$$\begin{cases} \nabla\times\mathbf{H} = \mathbf{J}, \\ \nabla\cdot(\mu\mathbf{H}) = 0, \end{cases} \tag{2}$$

where $\mathbf{J}$ is the current density (enforced current density $J_s\mathbf{e}_\theta$ in the coil, null elsewhere) and μ the magnetic permeability.

The Navier-Stokes equations are

$$\left\{ \begin{array}{l} \frac{D\mathbf{u}}{Dt} + \frac{1}{\rho}\nabla p - \nu\Delta\mathbf{u} = \alpha g(T - T_0)\mathbf{e}_z + \frac{\mu_0}{\rho} M\nabla H, \\ \nabla\cdot\mathbf{u} = 0, \end{array} \right. \quad (3)$$

where $\mathbf{u}$ is the velocity, p the pressure, T the temperature, T_0 the exterior temperature, ρ the density, ν the kinematic viscosity and α the thermal expansion. The last term of the right hand side represents the simplified expression of the Kelvin force in (1) considering that $\mathbf{M}$ and $\mathbf{H}$ are collinear [1]. The conservation of energy is

$$\rho c \partial_t T + \rho c(\mathbf{u}\cdot\nabla)T - \nabla\cdot(\lambda\nabla T) = f_T, \quad (4)$$

where c is the heat capacity, λ the thermal conductivity and f_T the heat source, equal to the Joule effect J_s^2/σ_{cl} in the coil and null elsewhere (σ_{cl} coil electrical conductivity).

The boundary conditions are: $\mathbf{u} = 0$ on the border of the fluid domain, $T = T_0$ and $\mathbf{H}\times\mathbf{n} = 0$ on the tank's exterior border. Initially, $\mathbf{u} = 0$, $T = T_0$, and $\mathbf{H} = 0$.

The magnetization intensity is proportional to the magnetic field intensity:

$$M = \chi(T)H, \quad (5)$$

with χ the susceptibility given by an approximation of Langevin's law:

$$\chi(T) = \frac{\phi\mu_0\pi d^3 M_0^2}{18 k_B T}, \quad (6)$$

where ϕ is the volume fraction, d the particle diameter, M_0 the particle magnetization and k_B the Boltzmann constant.

III. Physical Data

The dimensions are $H_t = 4$ cm, $R_t = 3$ cm, $R_i = 0.9$ cm, $R_e = 1.1$ cm, $e = 0.2$ cm, $L_0 = 2$ cm, and $e_w = 0.5$ cm.

The thermo-physical properties are different in each sub-domain (coil, fluid, tank). In these numerical experiments, the regular oil and the ferrofluid have the same properties to highlight the Kelvin force effect.

The exterior temperature is $T_0 = 293$ K. The ferrofluid characteristics are $\phi = 0.07$, $d = 10$ nm and $M_0 = 446$ kA/m. The enforced current is of 10 A and the corresponding current density is $J_s = 10^7$ A/m^2. For magnetostatic equations (2), the magnetic permeability is $\mu = \mu_0$ everywhere for simplicity.

IV. Results

Tests with analytical solutions exhibit the theoretical convergence rates and validate the numerical developments.

The code uses cylindrical coordinates. Every field f is solved as a partial Fourier sum relative to the azimuthal direction:

$$f(r,\theta,z) = \sum_{m=0}^{m_{max}} f_m^c(r,z)\cos(m\theta) + f_m^s(r,z)\sin(m\theta), \quad (7)$$

where f_m^c and f_m^s are approximated by FE. Computations show that, for the chosen parameters, the solution is carried by the mode 0 only, i.e. is axisymmetric. Even initially stimulated, the other modes rapidly vanish and $m_{max} = 0$ is thus chosen.

The mesh contains 2783 nodes. A time step of 0.0125 s is used over $4{\cdot}10^5$ iterations (18 wall-clock hours using 5 processors on the cluster IBM x3750-M4 from GENCI-IDRIS).

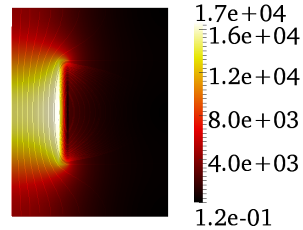

Fig. 2. Magnetic field intensity H (A/m) and field lines in a meridian plane (symmetry axis on the left) for regular oil and ferrofluid.

The system reaches a steady regime in about 1 h. The magnetic field (Fig. 2) impacts the ferrofluid flow. Because of the Kelvin force, a strong upward flow appears at the symmetry axis and leads to an additional convection cell that cools down the coil, as shown in Fig. 3.

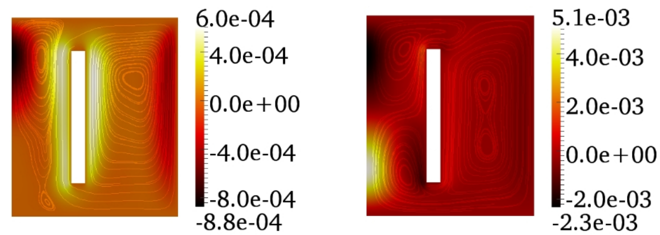

Fig. 3. Axial velocity u_z (m/s) and current lines in a meridian plane (symmetry axis on the left) at $t = 5000$ s for regular oil (left) and ferrofluid (right).

The magnetoconvection improves the heat removal in the ferrofluid system. The temperature increment $T - T_0$ in the coil goes from 34.4 K with regular oil to 30.7 K with ferrofluid, see Fig. 4. Thanks to the nanoparticles, the increase of temperature in the coil has been reduced by about 11 %.

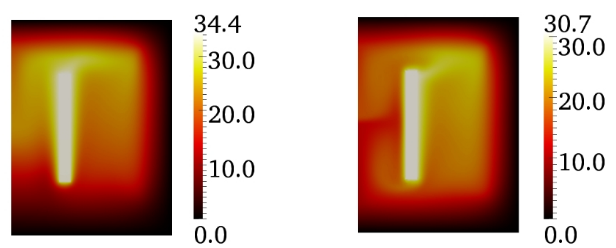

Fig. 4. Temperature increment $T - T_0$ (K) in a meridian plane (symmetry axis on the left) at $t = 5000$ s for regular oil (left) and ferrofluid (right).

References

[1] J. L. Neuringer, and R. E. Rosensweig, "Ferrohydrodynamics," *The Physics of Fluids*, vol. 7, no. 12, pp. 1927-1937, Dec. 1964.

[2] G.-Y. Jeong, S. P. Jang, H.-Y. Lee, J.-C. Lee, S. Choi, and S.-H. Lee, "Magnetic-Thermal-Fluidic Analysis for Cooling Performance of Magnetic Nanofluids Comparing With Transformer Oil and Air by Using Fully Coupled Finite Element Method," *IEEE Trans. Magn.*, vol. 49, no. 5, pp. 1865-1868, May 2013.

[3] L. Pîslaru-Danescu, A. M. Morega, G. Telipan, M. Morega, J. B. Dumitru, and V. Marinescu, "Magnetic Nanofluid Applications in Electrical Engineering," *IEEE Trans. Magn.*, vol. 49, no. 11, pp. 5489-5497, Nov. 2013.

[4] J.-L. Guermond, R. Laguerre, J. Léorat, C. Nore, "Nonlinear magnetohydrodynamics in axisymmetric heterogeneous domains using a Fourier/finite element technique and an interior penalty method," *J. Comp. Physics*, vol. 228, pp. 2739-2757, 2009.

[5] R. E. Rosensweig, "Magnetic Fluids," *Ann. Rev. Fluid Mech.*, vol. 19, pp. 437-63, 1987.